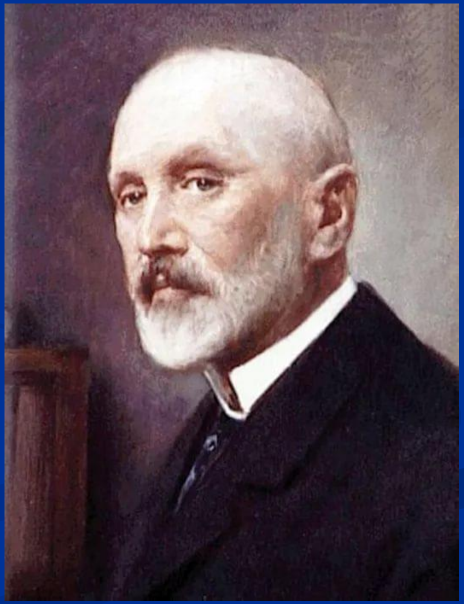
*1-st ISPMP-2019*
*Wuhan, HUST*
*29 April 2019*

# Measurements of the Newtonian gravitational constant: from Cavendish to Eötvös and to nowadays

Vadim Milyukov [1, 2]

*Lomonosov MSU*
*Sun Yat sen University*

Henry Cavendish (1731–1810),



Loránd Eötvös (1848-1919)

1EÖTVÖS
www.eotvos100.hu

UNESCO
United Nations
Educational, Scientific and
Cultural Organization

100th anniversary of Roland Eötvös (1848-1919), physicist, geophysicist, and innovator of higher education
Commemorated in association with UNESCO

| | |
|---|---|
| Subject: | Invitation to Eötvös 100 Honorary Board |
| Reference: | 626/2019/NKF |


**Prof. VadimMilyukov**
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*Budapest, 23 January, 2019*

**Dear Professor Vadim Milyukov,**

In association with UNESCO, the world's scientific community commemorates this year the 100$^{th}$ anniversary of the death of Roland Eötvös (1848-1919), a pioneer of high precision gravitational physics, founding father of geophysics and innovator of higher education.

Throughout the centenary year a series of scientific events and exhibitions will be organized, and special publications will be presented in Hungary and worldwide, all in remembrance of the genius of Baron Eötvös. A central event will take place in Budapest on the day of his passing away (8 April), and at the same time an "Eötvös 100" exhibition will open at the 2019 Annual Meeting of the European Geosciences Union, to be held in Vienna. Roland Eötvös will also be remembered at the General Assembly of the International Union of Geodesy and Geophysics (IUGG), an organization which also celebrates its 100th anniversary in 2019.

# Cavendish experiment 1797-1798

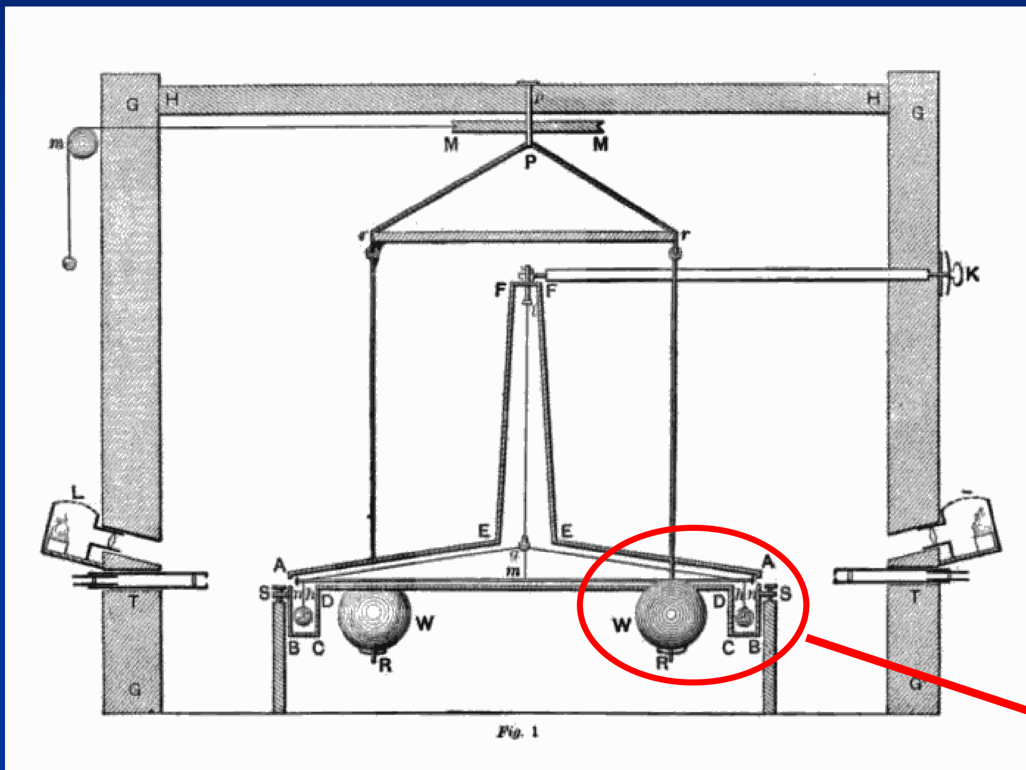
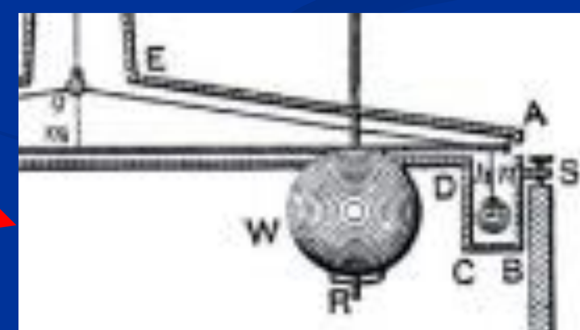



*Fig. 1*




- Wooden rod 1,8 m
- Spherical lead test masses 0.73 kg, D=51 mm
- Spherical lead attracting masses 158 kg, D= 300 mm

# Cavendish experiment

## Earth's density:

$\sigma = (5.448 \pm 0.033)\ \mathrm{g\,cm^{-3}}$

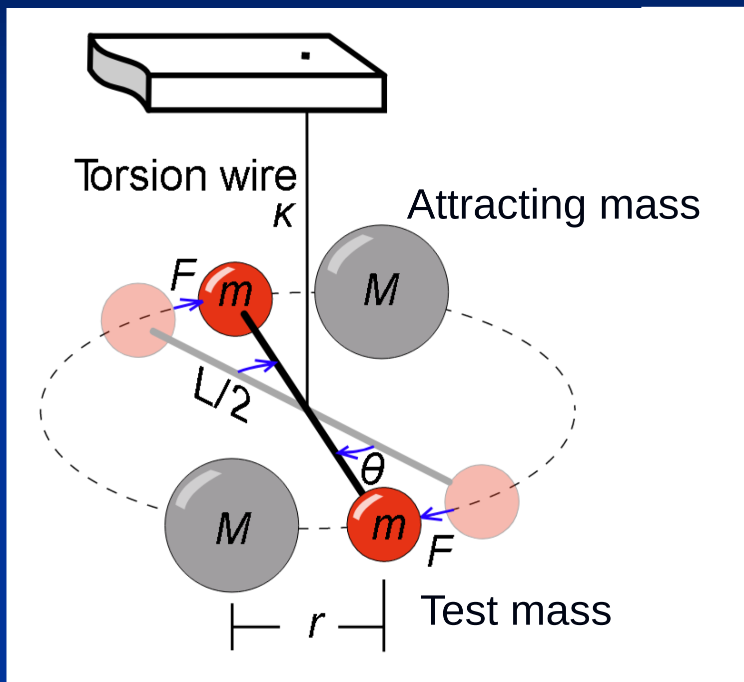
Cavendish, H. *Phil. Trans. R.Soc. B* **88**, 469–526 (1798).

## First G value:

$G = (6.67 \pm 0.07) \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$

$\Delta G/G \approx 10^4$ ppm

# Time – of – swing method




$$J\phi + [k + G(\partial\Gamma / \partial\varphi)]\varphi = 0$$

$$\omega_1^2 = \frac{k + G(\partial\Gamma / \partial\varphi)}{J}$$

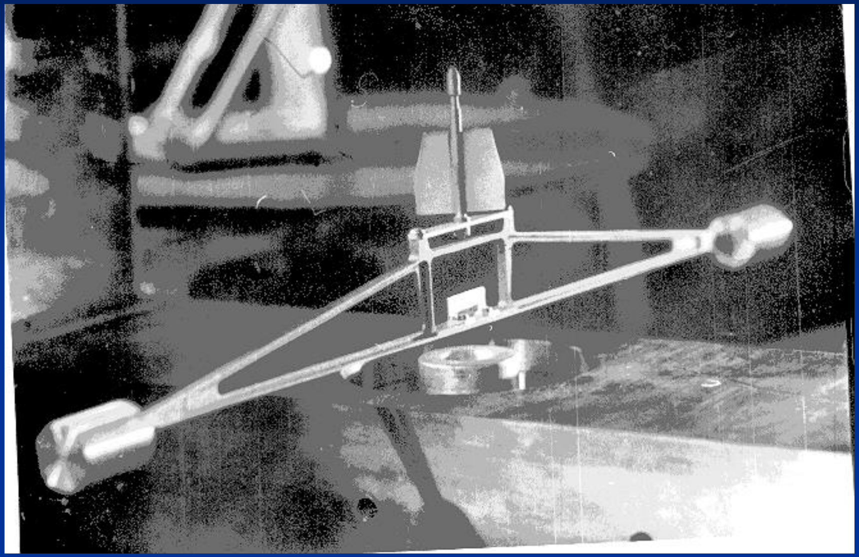
$$G = \frac{J\left[\left(\omega_1^2\right)_1 - \left(\omega_1^2\right)_2\right]}{\left(\partial\Gamma / \partial\varphi\right)_1 - \left(\partial\Gamma / \partial\varphi\right)_2}$$

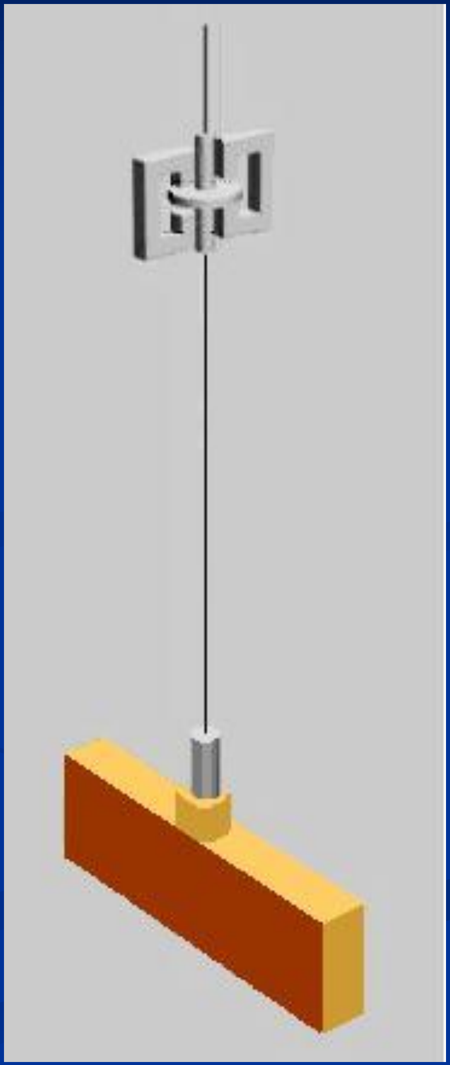
To determine G by time-of swing method it is necessary to perform the following measurements:

- frequencies of torsional oscillations at two different positions of the attracting masses M;
- geometric dimensions and densities of the torsion balance;
- geometric dimensions and density of the attracting mass M;
- the mutual location of the torsion balance and test masses M.

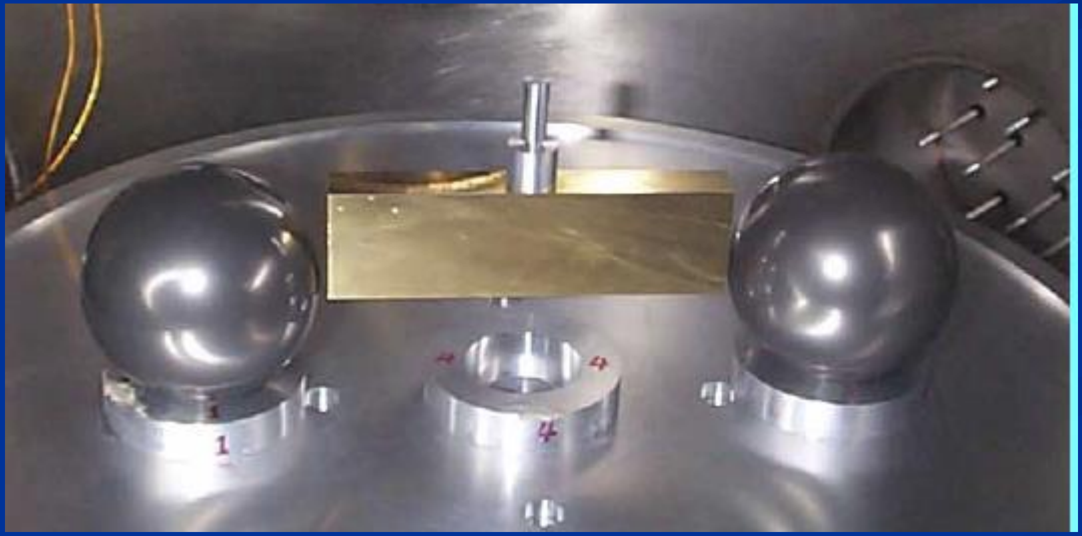
# The torsion balances and time of swing method

No 1

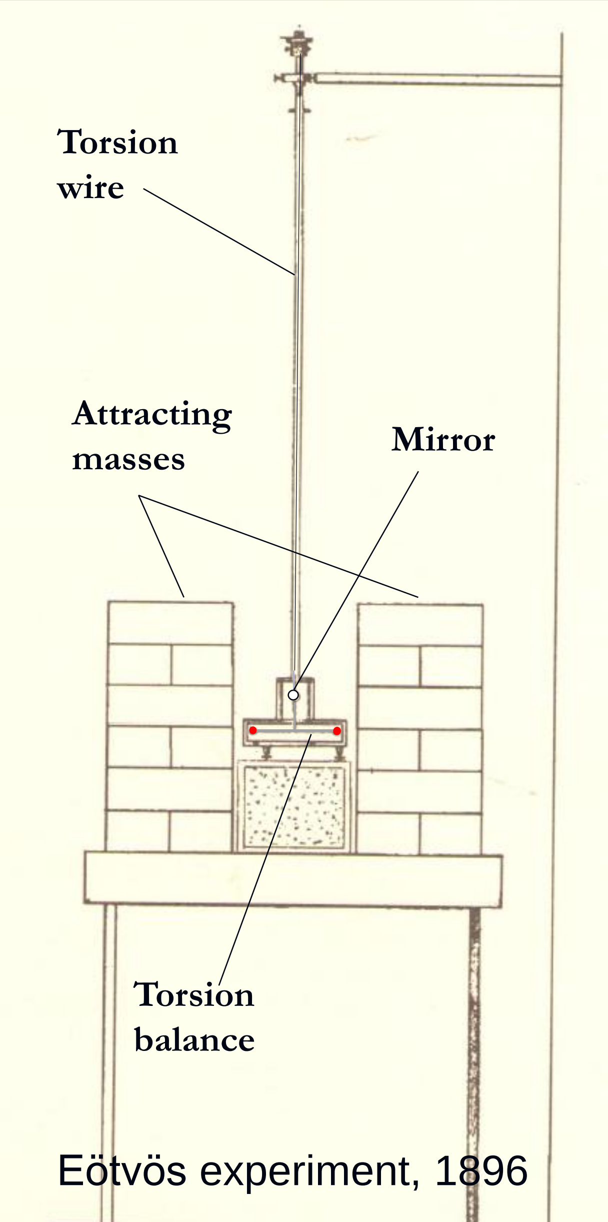




No 2



# Eötvös experiment




Eötvös experiment, 1896

The attracting masses: two lead parallelepipeds with a base of 30x30 cm and a height of 60 cm

Period of oscillation in the longitudinal position: $T1=641s$;
Period of oscillation in the transversal position: $T2=860$ s.

$$\left(\frac{1}{T_1^2}-\frac{1}{T_2^2}\right)=G\frac{8\rho}{\pi}(1-\varepsilon)$$

## Eötvös G value:

$$G=(6.657\pm0.013)\times10^{-11}\ m^3kg^{-1}s^{-2}$$

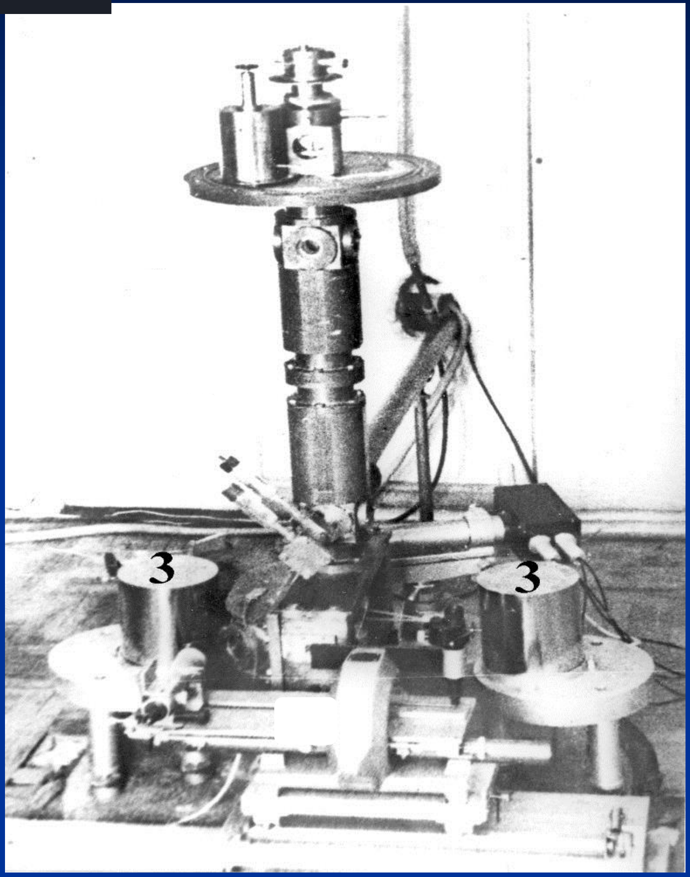
$$\Delta G/G=2\times10^3\ ppm$$

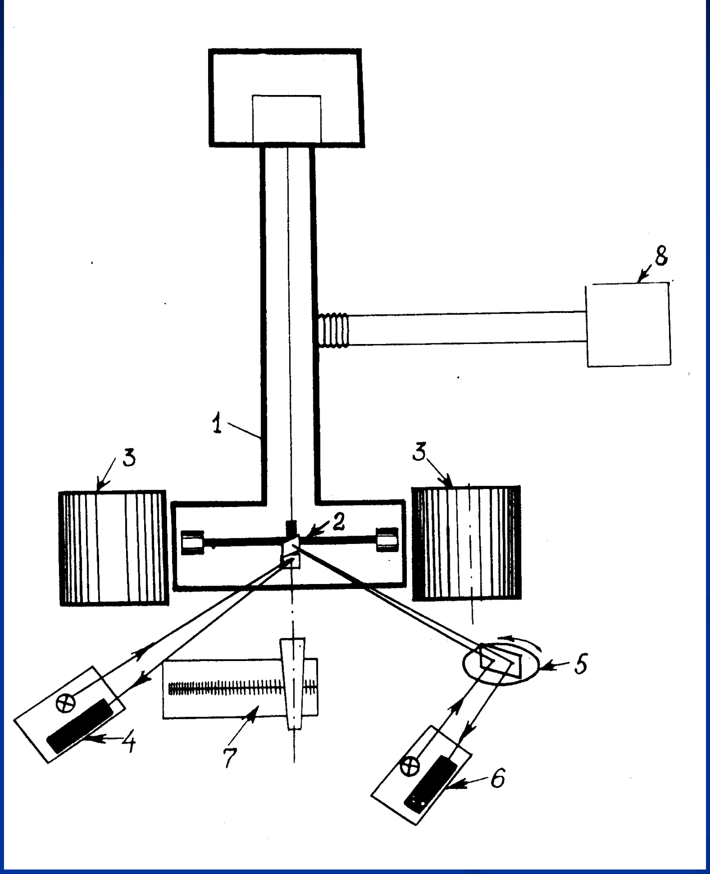
Lorand Eötvös. *Annalen der Phys. und Chem., N.F.*, **59**, 354-400 (1896)

# The most reliable measurements of the Newtonian gravitation constant (1 -2 stage)

| Experiment:<br>Author, year of publication,<br>Place of experiment | Technique | $G \times 10^{-11}\ m^3kg^{-1}s^{-2}$ |
|---|---|---|
| **H. Cavendish, 1798**<br>**Clapham, England** | Torsion balance, static | 6.67÷0.07 ~$10^4$ ppm |
| **F. Reich, 1838, 1852**<br>**Freiburg, Germany** | Torsion balance, dynamic | 6.64÷0.06 |
| **R. Eötvös, 1896**<br>**Budapest, Hungary** | Torsion balance, dynamic | 6.657÷0.013 |
| **C. Braun, 1897**<br>**Marienstein, Austria** | Torsion balance,<br>static and dynamic | 6.649÷0.02 |
| **P.Heil, 1930**<br>**Washington, USA** | Torsion balance, dynamic | 6.670÷0.005 ~$10^3$ ppm |
| **P.Heil, Chrzanowski, 1942**<br>**Washington, USA** | Torsion balance, dynamic | 6.673÷0.005 |
| **R. Rose, H.Parker, K. Lowry,**<br>**A. Kuhlhan, I. Beams, 1969**<br>**Charlotenscille, USA** | Torsion balance, compensative | 6.674÷0.004 |
| **J. Renner, 1973**<br>**Budapest, Hungary** | Torsion balance, dynamic | 6.670÷0.008 |

# SAI MSU experiment (1975-1978)




**General view and sketch of the SAI MSU experimental setup:**
1 – housing of device; 2 – torsion balance; 3 – attracting masses; 4, 6 – optical readout systems; 5 – rotating mirror; 7 – device for linear measurements; 8 – vacuum pump.

# The value of the Newtonian gravitational constant, obtained in the MSU experiment

Four series of the experiments:

1. March - May 1975,
2. March - June 1976,
3. January - March 1977
4. October 1977 – January 1978

$$G=(6.6745 \pm 0.0008)\times 10^{-11}\ m^3 kg^{-1} s^{-2}$$

*with relative standard uncertainties of 120 ppm*

Сагитов М.У, Милюков В.К. и др. Новое определение кавендишевой гравитационной постоянной// Доклады АН СССР, 1979, т.245, № 3, с.567 -569 (Sov. Phys. Dokl. **245**(1-6), 20-22 (1981))

# First HUST experiment (1998)

$G=(6.6699 \pm 0.0007)\times10^{-11}\ m^3kg^{-1}s^{-2}$ with rel. unc.: 105 ppm

Luo, J. et al. *Phys. Rev. D* **59**, 042001 (1998).

# Second HUST experiment (2009)



$G = (6.67349 \pm 0.00018) \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

with rel. unc.: 26 ppm

Luo, J. et al. *Phys. Rev. Lett.* **102**, 240801 (2009).

# The most reliable measurements of the Newtonian gravitation constant (3 -4 stage)

| Authors, year of publication | | Value of $G \times 10^{-11}$ $m^3 kg^{-1} s^{-2}$ | STD $\times 10^{-11}$ $m^3 kg^{-1} s^{-}$ | *ppm* |
|---|---|---|---|---|
| **Stage 3: ~100 ppm** | | | | |
| [1] *Facy and Ponticis,Ffrance* | 1972 | 6.6714 | 0.0006 | 90 |
| **[2] *Sagitov, Milyukov, et al. Moscow University*** | **1979** | **6.6745** | **0.0008** | **120** |
| [3] *Luther and Towler, Nat. Bur. of Stand., Washington* | 1982 | 6.6726 | 0.0005 | 75 |
| [4] *Michaelis, et al.,Physik Technische Bundesanstalt* | 1995 | 6.7154 | 0.0006 | 90 |
| [5] *Karagioz, Izmailov, Committee of Standards, Moscow* | 1996 | 6.6729 | 0.0005 | 75 |
| [6] *Bagley and Luther, Los Alamos National Lab* | 1997 | 6.6740 | 0.0007 | 105 |
| **[7] *Jun Luo, et al., HUST, China*** | **1999** | **6.6699** | **0.0007** | **105** |
| [8] *Fitzgerald and Armstrong Meas.St.Lab, New Zealnd* | 1999 | 6.6742 | 0.0007 | 105 |
| **Stage 4: ~15÷40 ppm** | | | | |
| [9] *Gundlach and Merkowich, University of Washington,* | 2000 | 6.674215 | 0.000092 | 14 |
| [10] *Quinn, Speake et all. University of Birmingham* | 2001 | 6.67559 | 0.00027 | 41 |
| [11] *Schlamminger et all. University of Zurich* | 2002 | 6.67407 | 0.00022 | 33 |
| [12] *Armstrong and Fitzgerald, Meas.St.Lab, New Zealnd* | 2003 | 6.67387 | 0.00027 | 40 |
| [13] *Schlamminger et all. University of Zurich* | 2006 | 6.674252 | 0.000109 | 16 |
| **[14] *Jun Luo, et al. HUST, China*** | **2009** | **6.67349** | **0.00018** | **26** |
| [15] *Parks and Faller, University of Colorado* | 2010 | 6.67234 | 0.00014 | 21 |

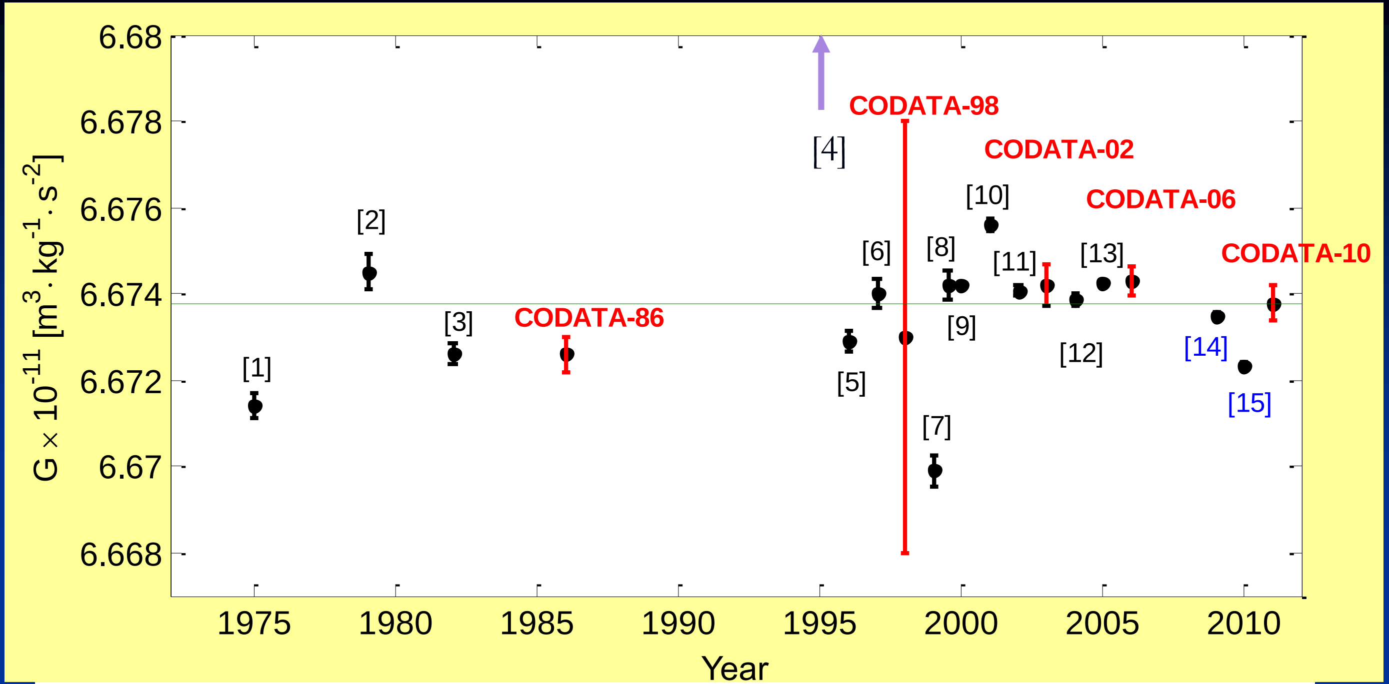
# CODATA values of the gravitational constant

The Task Group on Fundamental Constants of the Committee on Data for Science and Technology (CODATA) was established in 1969 to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all relevant data available at a given point in time.

CODATA - 1973 :
The recommended value of the Newtonian gravitation constant

$G=(6.6720 \pm 0.0041)\times 10^{-11}\ m^3 kg^{-1} s^{-2}$ (615 ppm)

was mainly based on the Heil and Chrzanowski results, obtained in 1942




| Authors, year of publication | Value of $G\times10^{-11}$ $M^3kg^{-1}s^{-2}$ | STD $\times10^{-11}$ $m^3kg^{-1}s^{-}$ | ppm |
|---|---|---|---|
| [1] *Facy and Ponticis, France* 1972 | 6.6714 | 0.0006 | 90 |
| [2] *Sagitov, Milyukov, et al. Moscow University* 1979 | 6.6745 | 0.0008 | 120 |
| [3] *Luther and Towler, Nat. Bur. of Stand., Washington* 1982 | 6.6726 | 0.0005 | 75 |
| **CODATA 1986** | **6.67259** | **0.00085** | **128** |
| [4] *Michaelis, et al., Physik Technische Bundesanstalt* 1995 | 6.7154 | 0.0006 | 90 |
| [5] *Karagioz, Izmailov, Committee of Standards, Moscow* 1996 | 6.6729 | 0.0005 | 75 |
| [6] *Bagley and Luther, Los Alamos National Lab* 1997 | 6.6740 | 0.0007 | 105 |
| **CODATA 1998** | **6.673** | **0.010** | **1500** |
| [7] *Jun Luo, et al., HUST, China* 1999 | 6.6699 | 0.0007 | 105 |
| [8] *Fitzgerald and Armstrong Meas.St.Lab, New Zealnd* 1999 | 6.6742 | 0.0007 | 105 |
| [9] *Gundlach and Merkowich, University of Washington,* 2000 | 6.674215 | 0.000092 | 14 |
| [10] *Quinn, Speake et all. University of Birmingham* 2001 | 6.67559 | 0.00027 | 41 |
| [11] *Schlamminger et all. University of Zurich* 2002 | 6.67407 | 0.00022 | 33 |
| **CODATA 2002** | **6.6742** | **0.0010** | **150** |
| [12] *Armstrong and Fitzgerald, Meas.St.Lab, New Zealnd* 2003 | 6.67387 | 0.00027 | 40 |
| [13] *Schlamminger et all. University of Zurich* 2006 | 6.674252 | 0.000109 | 16 |
| **CODATA 2006** | **6.67428** | **0.00067** | **100** |
| [14] *Jun Luo, et al. HUST, China* 2009 | 6.67349 | 0.00018 | 26 |
| [15] *Parks and Faller, University of Colorado* 2010 | 6.67234 | 0.00014 | 21 |
| **CODATA 2010** | **6.67384** | **0.00080** | **120** |

# Effect of unelasticity of torsion fiber (Kuroda effect)

Time- of –swing method

$$G = \frac{I(\omega_n^2 - \omega_f^2) - (k_n - k_f)}{C_{gn} - C_{gf}} = \frac{I\Delta(\omega^2)}{\Delta C_g}$$

In ideal case $K_n = k_f = const$

$$(k_n - k_f) = 0$$

In general case, spring constant *k* is a complex value, and not only dissipation,
but also *k* depends on the frequency

$$G = \frac{I(\omega_n^2 - \omega_f^2) - (k_n - k_f)}{C_{gn} - C_{gf}} = \frac{I\Delta(\omega^2)}{\Delta C_g}\left[1 - \frac{\Delta k}{I\Delta(\omega^2)}\right]$$

$$\Delta G/G \approx 1/\pi Q$$

For $Q \approx 1700$, ⇒ $\Delta G/G \approx 187$ ppm;
For $Q \approx 10^5$, ⇒ $\Delta G/G \approx 3$ ppm

Ways to overcome Kuroda effect :

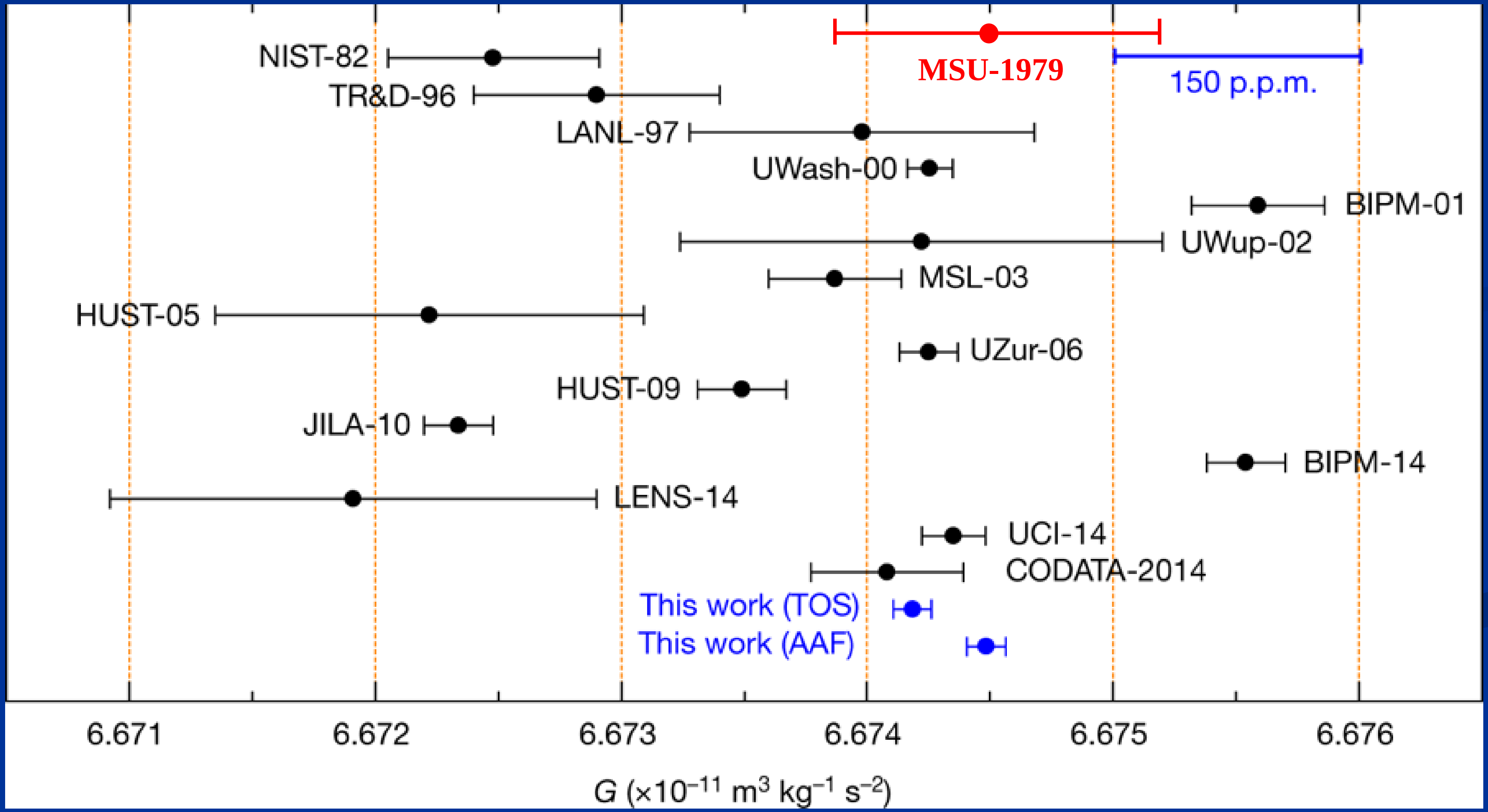
- Torsional balance with high Q
- The method of the experiment, in which the torsion wire is not twisted

- Time-of-swing (TOS) method;
- Angular-acceleration-feedback method (AAF)

# The most reliable measurements of the Newtonian gravitation constant

Luo Jun, et al (TOS) $G=(6.674184(78)\times10^{-11}\ m^3kg^{-1}s^{-2}$ with rel. unc.: 11.64 ppm

Luo Jun, et al (AAF) $G=(6.674484(78)\times10^{-11}\ m^3kg^{-1}s^{-2}$ with rel. unc.: 11.61 ppm



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doi.org/10.1038/s41586-018-0431-5

ARTICLE

https://doi.org/10.1038/s41586-018-0431-5

# Measurements of the gravitational constant using two independent methods

Qing Li[1,8], Chao Xue[2,3,8], Jian-Ping Liu[1,8], Jun-Fei Wu[1,8], Shan-Qing Yang[1]*, Cheng-Gang Shao[1]*, Li-Di Quan[4], Wen-Hai Tan[1], Liang-Cheng Tu[1,2], Qi Liu[2,3], Hao Xu[1], Lin-Xia Liu[5], Qing-Lan Wang[6], Zhong-Kun Hu[1], Ze-Bing Zhou[1], Peng-Shun Luo[1], Shu-Chao Wu[1], Vadim Milyukov[7] & Jun Luo[1,2,3]*

**The Newtonian gravitational constant, *G*, is one of the most fundamental constants of nature, but we still do not have an accurate value for it. Despite two centuries of experimental effort, the value of *G* remains the least precisely known of the fundamental constants. A discrepancy of up to 0.05 per cent in recent determinations of *G* suggests that there may be undiscovered systematic errors in the various existing methods. One way to resolve this issue is to measure *G* using a number of methods that are unlikely to involve the same systematic effects. Here we report two independent determinations of *G* using torsion pendulum experiments with the time-of-swing method and the angular-acceleration-feedback method. We obtain *G* values of $6.674184 \times 10^{-11}$ and $6.674484 \times 10^{-11}$ cubic metres per kilogram per second squared, with relative standard uncertainties of 11.64 and 11.61 parts per million, respectively. These values have the smallest uncertainties reported until now, and both agree with the latest recommended value within two standard deviations.**

# Conclusion

1798

Henry Gavendish : “The apparatus is very simple”
(*Philos. Trans. R. Soc. London,* ***88****, 469, 1798)*

2010

James Faller:

“Big G is the Mt. Everest of precision measurement science, and it should be climbed.”
(*Phys. Rev. Lett.*, ***105****, 2010)*

2018

Stefan Schlaminger:

“Gravity measured with record precision”
*562 (Nature, 560, 2018)*