Weakly nonlocal thermodynamics and Newtonian gravitation

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Barcelona, 21/05/2019

Collaboration with Sumiyoshi Abe and Robert Kovács

Outline

1 About the origin of evolution equations

2 Extensivity

3 Newtonian gravitation

4 Eötvös 100

Evolution with dissipation

Variational principles

- Ideal materials without dissipation
- Dissipation potentials? Gyarmati principle?
- There are many different variational principles for dissipative processes

Mixed strategies

- Phase fields, Ginzburg-Landau
- GENERIC

Pure thermodynamics?

- The heuristic road: thermostatics and the separation of divergences
- The rigorous way: Coleman-Noll and Liu procedures
- Rigorously blocked? Stability and material frame indifference

Methods from thermodynamics

Pure thermodynamics? Extended and internal variables

- The heuristic road: thermostatics and the separation of divergences
- The rigorous way: Coleman-Noll and Liu procedures. Blocked?

Benchmarks

- Ideal processes, variational principles
- Material frame indifference, the aspect of spacetime
- Weak nonlocality
- The problem of inertia

Generalizations

- Entropy flux is constitutive
- Derivative of a constraint can be a constraint
- Extensivity

Methods from thermodynamics

Pure thermodynamics?

- The heuristic road: thermostatics and the separation of divergences
- The rigorous way: Coleman-Noll and Liu procedures.

Benchmarks

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Thermostatics, the hidden knowledge

Berezovski-Ván (Springer, 2017)
Biró et al. PLB, 2018.

Thermostatics of fluids: $S(E, V, M)$

From discrete to continuum: extensivity.

$\lambda S(E, V, M) = S(\lambda E, \lambda V, \lambda M) \leftrightarrow \exists s(e, v) \leftrightarrow E = TS - pV + \mu M$

Gibbs relation for fluids: specific quantities, $s(e, v)$

$$de = Tds - pdv = Tds + \frac{p}{\rho^2} d\rho, \qquad e = Ts - pv + \mu.$$

Gibbs relation for fluids: densities, $\rho_s(\rho_e, \rho)$

$$\rho_e = \rho e, \qquad \rho_s = \rho s, \qquad \rho = 1/v$$

$$d\rho_e = Td\rho_s + \mu d\rho, \qquad \rho_e = T\rho_s + \mu\rho - p.$$

Thermostatics of elasticity: $S(E, \epsilon, \rho)$?

Deformation is not extensive.

Gibbs relation for elasticity: specific quantities, $s(e, \epsilon)$

$$de = Tds + \boxed{\frac{\sigma}{\rho}} : d\epsilon, \qquad e = Ts + \frac{\sigma}{\rho} : \epsilon + \mu.$$

Gibbs relation for elasticity: densities, $\rho_s(\rho_e, \epsilon)$

$$d\rho_e = Td\rho_s + \boxed{\sigma} : d\epsilon + \left(\mu + \frac{\sigma : \epsilon}{\rho}\right) d\rho, \qquad \rho_e = T\rho_s + \sigma : \epsilon + \mu\rho.$$

Gibbs relation for elasticity: free energy density, $\rho_f(T, \epsilon, \rho)$

$$d\rho_f = -\rho_s dT + \sigma : d\epsilon + \left(\mu + \frac{\sigma : \epsilon}{\rho}\right) d\rho, \qquad \rho_f = \sigma : \epsilon + \mu\rho.$$

A fluid with a scalar internal variable: $s(e, \rho, \varphi, \nabla\varphi)$

Gibbs relation: specific quantities $s(e, \rho, \varphi, \nabla\varphi)$

$$du = Tds + \frac{p}{\rho^2} d\rho = de - d\left(\varphi + \frac{\nabla\varphi \cdot \nabla\varphi}{8\pi G\rho}\right).$$

Balances of mass, momentum and internal energy:

$$\dot{\rho} + \rho\nabla \cdot \mathbf{v} = 0,$$
$$\rho\dot{\mathbf{v}} + \nabla \cdot \mathbf{P} = \mathbf{0},$$
$$\rho\dot{e} + \nabla \cdot \mathbf{q} = -\mathbf{P} : \nabla\mathbf{v}.$$

$$\boxed{\rho\dot{s} + \nabla \cdot \mathbf{J} = \Sigma \geq 0}$$

Method: separation of divergences: $\dot{s}(e, \rho, \varphi, \nabla\varphi)$ + balances.

Weakly nonlocal internal energy: $s(e, \rho, \varphi, \nabla\varphi)$

$$\dot\rho + \rho\nabla\cdot\mathbf{v} = 0, \quad \rho\dot e + \nabla\cdot\mathbf{q} = -\mathbf{P} : \nabla\mathbf{v}$$

$$\rho\dot s = \rho\left(\partial_e s\dot e + \partial_\rho s\dot\rho + \partial_\varphi s\dot\varphi + \partial_{\nabla\varphi}s(\nabla\varphi)^{\cdot}\right) =$$

$$= \frac{1}{T}\rho\dot e + \frac{p}{T}\frac{\dot\rho}{\rho} - \frac{\rho}{T}\dot\varphi - \frac{1}{8\pi G T\rho}(\nabla\varphi)^2\dot\rho - \frac{1}{4\pi G T}\nabla\varphi\cdot(\nabla\varphi)^{\cdot} =$$

$$\dots$$

$$= -\nabla\cdot\left[\frac{1}{T}\left(\mathbf{q} + \frac{\dot\varphi}{4\pi G}\nabla\varphi\right)\right]$$

$$+\left(\mathbf{q} + \frac{\dot\varphi}{4\pi G}\nabla\varphi\right)\cdot\nabla\left(\frac{1}{T}\right)$$

$$+\frac{\dot\varphi}{4\pi G T}\left(\Delta\varphi - 4\pi G\rho\right)$$

$$-\left[\mathbf{P} - p\mathbf{I} - \frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi - \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\mathbf{I}\right)\right] : \frac{\nabla\mathbf{v}}{T} \geq 0$$

Thermal, mechanical and gravitational thermodynamic fluxes and forces.

Dissipative gravitation?

Main result:

$$\dot{\varphi} = \frac{l_1}{T}\left(\frac{\Delta\varphi}{4\pi G} - \rho\right) - \frac{l_{12}}{T}\nabla\cdot\mathbf{v}.$$

Ideal gravitation: Poisson equation

$$\Delta\varphi = 4\pi G\rho,$$

$$\mathbf{P} = p\mathbf{I} + \frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi - \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\mathbf{I}\right)$$

Surface or volumetric?

$$\rho\dot{\mathbf{v}} + \nabla\cdot\mathbf{P} = \mathbf{0},$$

$$\nabla\cdot\left(\frac{1}{4\pi G}\left(\nabla\varphi\nabla\varphi - \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\mathbf{I}\right)\right) = \rho\nabla\phi,$$

$$\rho\dot{\mathbf{v}} + \nabla\cdot\mathbf{P}_{NS} = -\rho\nabla\phi.$$

Extensive or not extensive?

Modified temperature?

$$du = d\left[e_{tot} - \frac{v^2}{2} - \varphi - \frac{\nabla\varphi\cdot\nabla\varphi}{8\pi G\rho}\right] = Tds - pdv$$

Energy of gravitational field

density: $\rho_{fgrav} = \frac{(\nabla\varphi)^2}{8\pi G}$, specific: $e_{fgrav} = \frac{(\nabla\varphi)^2}{8\pi G\rho}$

Long range forces $\rightarrow$ not extensive.

From continuum to discrete: form dependence

$$\lambda S(E,V,M) = S(\lambda E, \lambda V, \lambda M) \leftrightarrow \exists s(e,v) \leftrightarrow E = TS - pV + \mu M$$

$$s(e,v,..) \overset{homogeneity}{\longleftrightarrow} \lambda S(E,V,M) = S(\lambda E, \lambda V, \lambda M)$$

Summary

Laws of physics from thermodynamics

- A world with many roads. Heuristic is good, rigorous is better.
- Extended Thermodynamics
- Internal variables, weak nonlocality

Are internal variables internal?

- Thermodynamic rheology $\to$ elasticity and local internal variable
- Phase fields (Cahn-Allen, Cahn-Hilliard) $\to$ weakly nonlocal internal variable and constraints
- Generalized continua (Cosserat, Eringen, Mindlin) $\to$ elasticity with weakly nonlocal dual internal variables
- Generalized heat conduction, rarefied gases $\to$ hierarchy of internal variables with increasing tensorial orders
- Korteweg and quantum fluids $\to$ fluid with weakly nonlocal density
- Thermodynamic gravitation $\to$ mechanics with weakly nonlocal energy

Gravitation challenge

Prediction and verification

- Thermodynamic rheology → confirmed (in rock experiments)
- Phase fields (Cahn-Allen, Cahn-Hilliard) → accepted
- Generalized continua (Cosserat, Eringen, Mindlin) → submitted
- Generalized heat conduction → accepted + confirmed (in room temperature experiments) + presentations of F. Vázquez, T. Fülöp, M. Szücs
- Rarefied gases → accepted + presentation of R. Kovács
- Korteweg and quantum fluids → accepted
- Thermodynamic gravitation → accepted (experiment??)

Gravitation: prediction

and

experimental verification??

Orosz István:
Eötvös Loránd

Eötvös Loránd

1848 – 1919

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Educational, Scientific and
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Egyesült Nemzetek
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Kulturális Szervezete

100th anniversary of Roland Eötvös (1848-1919), physicist, geophysicist, and innovator of higher education
Commemorated in association with UNESCO

Eötvös Loránd (1848-1919) fizikus, geofizikus és a felsőoktatás megújítójának 100. évfordulója
Az UNESCO-val közösen emlékezve

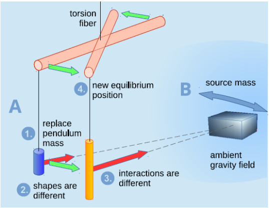
Eötvös balance

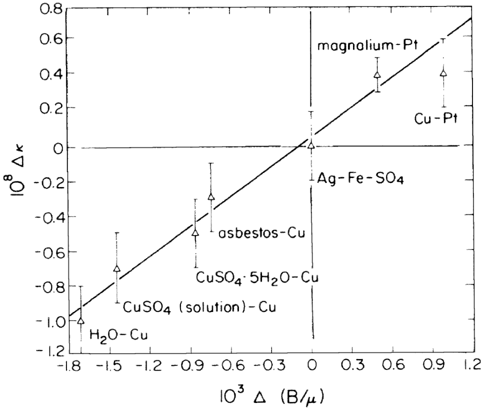
Sensitivity: $10^{-9} \to 10^{-11}$

Eötvös balance: the 5th force mystery

Eötvös ratio:

$$\eta = \Delta\kappa = 2\frac{|a_1 - a_2|}{|a_1 + a_2|} =$$
$$= \frac{m_{P1}}{m_{I1}} - \frac{m_{P2}}{m_{I2}} =$$
$$= C\Delta\left(\frac{B}{m}\right)$$





Fischbach et al. (1986)

Jánossy Underground Physics Laboratory

- Laboratory preparation is finished
- Motorization is ready
- Cameras are installed
- Precision asimuth readout is working
- Software for readout and evaluation is written
- Test masses are manufactured and inserted
- EP measurement has been started last week

c

Sensitivity : $10^{-9} \rightarrow 10^{-11}$

Thank you for your attention!